13.1: Permutations and Combinations

Isaac is a freshman at Kent State University. He is planning his fall schedule for next year. He has the choice of 3 math courses, 2 science courses and 2 humanities courses. He can only select one course from each area.

How many course schedules are possible?

- Independent events:

- Dependent events:

In the example of the course scheduling, does the choice of selecting a math course affect the choice of ways to select a science or humanities course?

What if the course times were included? Would this make the events independent or dependent?

The branch of mathematics that studies different possibilities for the arrangement of objects is called ________________.

- Basic Counting Principle: Suppose one event can be chosen in p different ways, and another independent event can be chosen in q different ways. Then the two events can be chosen successively in pq ways. Note: This can be extended to any number of independent events.
Examples:

1.) In this class, how many girl-boy pairs are possible?

2.) If you toss a coin, then roll a 6-sided die and then spin a 4-colored spinner with equal sections, how many outcomes are possible?

3.) How many ways are there to line up all the girls in this room?

Factorial Notation: \( n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 \)

Examples:

1.) \( 3! = \)

2.) \( 6! = \)

In the calculator: MATH - PRB - 4: !

The number of permutations of \( n \) objects, taken \( n \) at a time, is defined as \( P(n,n) = n! \).

Examples:

1.) How many ways are there to line up the boys in this class?

2.) There are 5 favorite runners in a race. How many ways can the runners win 1st, 2nd, 3rd, 4th and 5th place?
Example: There are ___ boys in this class. How many ways can we select the positions of Math Guru, Math Genius and Math Wizard, assuming that one person cannot hold more than one position?

This is a permutation of ___ objects taken 3 at a time. We need a new rule.

The number of permutations of n objects, taken r at a time, is defined as $P(n,r) = \frac{n!}{(n-r)!}$.

In the calculator: MATH - PRB - 2: nPr

Note: With permutations: ORDER MATTERS

Example: Suppose Mrs. Meinke wants to know the number of possible girl-girl pairs in this class. Why can't a permutation be used?

The number of combinations of n objects taken r at a time is defined as $C(n,r) = \frac{n!}{(n-r)!r!}$.

In the calculator: MATH - PRB - 3: nCr

Note: With combinations: ORDER DOES NOT MATTER.

Now, using combinations, let's find the number of girl-girl pairs.
13.2 Permutations with Repetitions and Circular Permutations

Examples:
1.) How many ways can the letters of the word MATH be arranged?

2.) How many ways can the letters of the word CALCULUS be arranged?

The number of permutations of $n$ objects of which $p$ are alike and $q$ are alike is $\frac{n!}{p!q!}$. Note: This can be extended to any amount of "like" objects.

3.) How many ways can the letters of the word MISSISSIPPI be arranged?

So far, we've been studying objects that are arranged in a line. When objects are arranged in a circle, some of the arrangements are alike.

Example: Consider the problem of making distinct arrangements of people sitting around a table playing cards. How many seating arrangements are possible?

If $n$ objects are arranged in a circle, then there are $n!$ or $(n-1)!$ permutations of the $n$ objects around the circle. $\frac{n!}{n}$

Example: How many ways are there to place 12 decorative symbols around the face of a clock?
13.3 Probability and Odds

What are the chances???

Examples:
1.) What is the probability that a student chosen at random in this class is female? Male?

Total number of students: Male: 23 Female: 11

\[ P(\text{female}) = \frac{11}{23} \]
\[ P(\text{male}) = \frac{12}{23} \]

2.) Let’s talk about the probability of a student chosen at random having a certain eye color.

- Green: 0
- Blue: 9
- Brown: 10
- Hazel: 4

\[ P(\text{green}) = \frac{0}{23} = 0 \]
\[ P(\text{not blue}) = \frac{14}{23} \]
\[ P(\text{not hazel}) = \frac{19}{23} \]

3.) A circuit board with 20 computer chips contains 4 chips that are defective. If 3 chips are selected at random, what is the probability that all 3 are defective?

There are \( \binom{4}{3} \) ways to choose 3 out of 4 defective chips and \( \binom{20}{3} \) ways to select 3 out of 20 chips.

\[ P(3 \text{ defective chips}) = \frac{\binom{4}{3}}{\binom{20}{3}} = \frac{4 \text{ ncr } 3}{20 \text{ ncr } 3} \]

4.) Paul has 10 rap, 18 rock, 8 country and 4 pop CDs in his music collection. Two are selected at random. Find each probability.
   a.) \( P(2 \text{ pop}) \)
   b.) \( P(\text{not rock}) \)
   c.) \( P(1 \text{ rap and 1 rock}) \)
13.4 Probabilities of Compound Events

If two events, A and B, are independent, then the probability of both events occurring is the product of each individual probability.

\[ P(A \text{ and } B) = P(A)P(B) \]

1.) Using a standard deck of playing cards, find the probability of selecting a face card, replacing it in the deck, and then selecting an ace.

\[ \frac{12}{52} \cdot \frac{4}{52} = \frac{3}{169} \]

If two events, A and B, are dependent, then the probability of both events occurring is the product of each individual probability.

\[ P(A \text{ and } B) = P(A)P(B \text{ following } A) \]

2.) Find the probability of selecting a face card, NOT replacing it in the deck, and then selecting an ace.

\[ \frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221} \]

Examples: Determine if the events are independent or dependent. Then determine the probability.

1.) the probability of randomly selecting two oranges from a bowl of 5 oranges and 4 tangerines, if the first selection is replaced. 

\[ \begin{align*} 
A &= \text{selecting an orange} \\
B &= \text{selecting an orange} \\
\text{Independent} \quad \frac{5}{9} \cdot \frac{5}{9} &= \frac{25}{81} 
\end{align*} \]

2.) A green number cube and a red number cube are tossed. What is the probability that a 4 is shown in the green number cube and a 5 is shown on the red number cube?

\[ \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \]

3.) the probability of randomly selecting a knife, a fork and a spoon in that order from a kitchen drawer containing 8 spoons, 8 forks and 12 table knives.

\[ \frac{12}{28} \cdot \frac{8}{27} \cdot \frac{8}{26} = \frac{32}{819} \]
-Mutually exclusive events:

Events that cannot happen at the same time.
Ex: Rolling a 2 and rolling a 6.

If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.
\[ P(A \text{ or } B) = P(A) + P(B) \]

1.) Find the probability of rolling a 2 or a 5 on a standard die.

\[ \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \]

2.) Find the probability of pulling an ace or a queen from a standard deck of cards.

\[ \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \]

-inclusive events:

If two events, A and B, are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Example Let’s roll two standard dice.
Let A = the first number cube shows a 2
Let B = the sum of the two number cubes is 6 or 7
Find P(A or B).