6.1 Roots and Radical Expressions

Name: __________________

Objectives: Students will be able to find nth roots.

3² = 9, so 3 is a square root of 9
3³ = 27, so 3 is a cube root of 27
3⁴ = 81, so 3 is a 4th root of 81
...
3ⁿ = ___, so 3 is an nᵗʰ root of ___.

Definition: If aⁿ = b, with a and b real numbers and n a positive integer, then a is an nth root of b.

Example - Even Root

\[ \sqrt{25} = 5 \] (principal root)

Example - Odd root

\[ \sqrt[3]{-8} = -2 \]

\[ \sqrt[4]{16} = 2 \]

\[ \sqrt{-1} = \text{imag} \]

Examples: Find all the real roots.

1.) \( \sqrt{-25} \)
2.) \( \sqrt[3]{-8} \)
3.) \( \sqrt[4]{16} \)
4.) \( -\sqrt[3]{27} \)
5.) \( \sqrt[2]{81/100} \)
6.) \( \sqrt[3]{-1} \)
7.) \( \sqrt[4]{0.027} \)
8.) \( \sqrt[3]{81/16} \)

\[ \sqrt{2^2} = 2 \]
\[ \sqrt{(-2)^2} = 2 \]
\[ \sqrt[3]{3^3} = 3 \]
\[ \sqrt[3]{(-3)^3} = -3 \]

Property: nth Roots of nth Powers

For any real number \( a \), \( \sqrt[n]{a^n} = \begin{cases} |a|, & n \text{ is even} \\ a, & n \text{ is odd} \end{cases} \)
Examples  Simplify each radical expression. Use absolute value symbols when needed.

1. $\sqrt[2]{16x^2} = 4|x|$

2. $\sqrt[2]{27y^6} = 3y^3$

3. $\sqrt[20]{x^{20},28} = |x^5 y^7|$

4. $\sqrt[20]{32y^{10}} = 2y^5$

5. $\sqrt[12]{x^{12},16} = \left| \frac{x^2 y^3}{y^3} \right| = x^2 |y^3|$

* If the root is even and the resulting exponent is even, the absolute value can be omitted.

Electricity  The voltage $V$ of an audio system’s speaker can be represented by $V = 4\sqrt{P}$, where $P$ is the power of the speaker. An engineer wants to design a speaker with 100 watts of power. What will the voltage be?

Find the power if the voltage is 40 volts.

$V = 40$

$40 = 4\sqrt{P}$

$16 = P$

$P = 100$